CSMA 2022
15ème Colloque National en Calcul des Structures
16-20 Mai 2022, Presqu'île de Giens (Var)

Involuntary tremor control using passive non-smooth absorber

S. Gebai, A. Ture Savadkoohi, C.-H. Lamarque

LTDS UMR CNRS 5513, École Nationale des Travaux publics de l'État,
{sarah.souheilgebai,alireza.turesavadkoohi,claude-henri.lamarque}@entpe.fr

Résumé — Patients affected by a pathological tremor, at the level of their upper limbs, face an abnormal involuntary movement disorders. Until now, no medication is found to cure the disease and control the tremor completely. In this paper, a non-linear energy sink with a non-smooth function is suggested as a mechanical solution to control the tremor. The proposed design procedure is based on the detection of fast and slow dynamics of the system. Analytical developments are compared with numerical results showing that analytical tests can be used to predict the response of the system.
Mots clés — Essential tremor, passive control, non-smooth absorber.

1 Introduction

Patients with pathological tremor suffer from an involuntary tremor which results from antagonistic muscle contractions [1]. It can affect different body parts, like upper limbs, legs, and/or trunk, etc. Pathological tremor can be categorized into rest tremor (RT), postural tremor (PT), and kinetic tremor (KT), where patients can have one or several types of tremors. Involuntary tremor presented in fully supported upper limbs, with no voluntary muscle contraction, is referred as the RT and characterized by a frequency range of 3-7 Hz [2]. PT refers to upper limbs maintained voluntarily at a specific position against gravity, and it is characterized by frequencies that lies between 5-12 Hz [2]. KT represents more daily tasks and appears with any voluntary movement, its frequency range is 3-10 Hz [3]. Parkinson disease (PD) is related to the RT and PT, whereas essential tremor (ET) is a bilateral of PT and KT.

ET can lead patients to become socially isolated, since they will feel embarrassed to face people due to difficulties in performing the daily life tasks, like eating, drinking, and writing, etc. Medications used to deal with such a problem can decrease the tremor progress for a period of time, but can not treat the disease. In addition, the given doses of medication will be gradually increased over time and can cause addiction [4]. On the other hand, some patients might not respond to these medications, so the brain stereotactic surgery, like the deep brain stimulation (DBS), can be applied.

Medications and surgical treatments can deteriorate the life quality of the patients and lead to severe side effects. Research studies shifts to the mechanical solution, using vibration controllers, to reduce the tremor vibrational movement visualized at the upper limbs of the patients. In the patent of Rosan [5], he suggested the use of active systems to control the pathological tremor and avoid medications. A chamber including viscous fluid is used to provide a linear velocity-dependent force based on rotation-sensing actuators and a generation of electrical signals. This device was used to mitigate the tremor in four directions, three translational and one rotational movement. Active controllers use feedback information of the sensors and provide an instantaneous tremulous control force. Several researchers tried to develop in the active type of tremor controllers without reaching a high level of vibration reduction [6]. However, the main problem remains that the active controllers has stability problems and complex design, in addition to the high activation power requirements.

Passive vibration controllers can be interesting due to the simplicity in their design with no power requirements for operation. Hashemi et al. [7] suggested the replacement of active by passive linear controller to reduce the RT for patients with PD. A tuned vibration absorber of 130 g mass is used to reduce the flexion-extension angular displacement response of a two degrees-of-freedom (DOF) upper limb modelled in the horizontal plane, where the numerical and experimental results show to be qualitatively similar. Gebai at el. [8] used three tuned mass dampers (TMD) of 15.7% total mass ratio to reduce the PT of a patient with ET. The effectiveness of this TMD-system, in the tuned and optimized cases, is tested on an experimental-arm excited by Electromyography signal of an affected patient. The proposed passive linear controller causes a significant reduction in the amplitude of the system at the dominant peak frequency. However, the TMDs causes an amplification in the amplitude of the system at a new frequency referred to the coupled system (experimental-arm with TMDs). The main disadvantage of the passive linear controllers is their sensitivity to changes in tremor frequencies and the narrow operational bandwidth, in addition to loading the system. Developments are required in the passive linear controller since the tremor frequency is investigated to shift within a range of 1 Hz wide [9].

Ture Savadkoohi et al. [10] proposed the use of passive non-linear absorbers as a solution to provide a tremor amplitude reduction at the upper limbs of patients with ET over a wide frequency bandwidth. The non-linear energy sink (NES) [11] of a cubic restoring forcing function is attached to a dynamic model of the upper limb, and its parameters are designed based on the time multiple scale method that leads to the detection of the fast and slow dynamics of the system. The NES of light mass was able to control the diverging response of the undamaped main system. In this paper, a NES of non-smooth function (piece-wise linear) is used, due to the existence of a gravitational field in the main system which is modelled as two links oscillating in the vertical plane, to obtain a better controllability results.

2 Non-linear dynamics

2.1 Modelled system

The upper limb of a human is modelled dynamically as two rigid links, the upper arm and forearm and hand as one segment, as shown in Figure 1. The mass of each segment is concentrated at the centroid of the link. The links are connected by rotational joints that permit the flexion-extension motion in the vertical plane. The movement of the modelled system is produced by three muscles, two signal joint shoulder and elbow muscles, and one double joint biceps brachii muscle [12]. The resistive torque of the muscle is produced by an elastic and viscous stiffness as provided by Jackson et al. [13]. The active torque at both single joints are modelled as sinusoidal signals operating at a frequency corresponding to the essential tremor of the patient suffering from the ET. It can be enough to consider an upper limb model oscillating in one direction since the other directions of motion (pronation-supination and radial-ulnar) represents a motion at nearly the same frequency [9]. The length, position of centroid, mass, and mass moment of inertia of each segment can be modelled as a percentage of the patient's body mass and height approximated by the equations deduced from experimental measurements by Harless [14]. The values of these parameters are shown in Table 1, where the indices 1 and 2 refers to the upper arm and forearm, respectively. θ_1 and θ_2 refer to the flexion-extension angular displacement motions at the shoulder and elbow joints, respectively, which need to be controlled by the NES of displacement u.

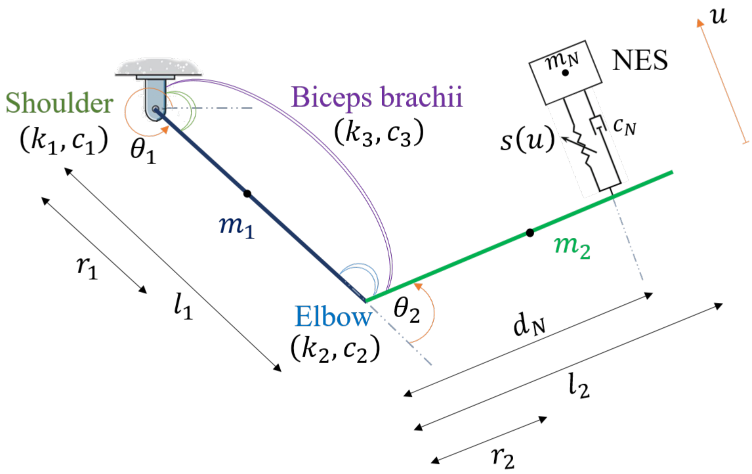

FIGURE 1 – Musculoskeletal upper limb model with NES attached to the forearm

The NES is added to the forearm segment at a chosen distance d_N of 26.9 cm away from the elbow

TABLE 1 – Upper limbs physical parameters indicated by Harless [14]

Mass (kg)	Length (cm)	Centroid (cm)
m_1=2.7	l_1=36.4	r_1=15.5
m_2=1.7	l_2=50.2	r_2=20.3

joint, as shown in Figure 1. It is expected to cause reduction for the angular displacements of other directions of motion as well [15]. The general form of the governing non-linear equation of motion of the system, derived using Lagrange formula [16], is obtained as :

$$M\begin{Bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\\ \ddot{u}\end{Bmatrix}+K\begin{Bmatrix}\theta_1\\ \theta_2\\ 0\end{Bmatrix}+C\begin{Bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{u}\end{Bmatrix}+\begin{Bmatrix}G_1\\ G_2\\ G_3\end{Bmatrix}+B\begin{Bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{u}\end{Bmatrix}-\begin{Bmatrix}\mathcal{T}_1\\ \mathcal{T}_2\\ 0\end{Bmatrix}+\begin{Bmatrix}0\\ 0\\ s(u)\end{Bmatrix}=\begin{Bmatrix}F_1\\ F_2\\ 0\end{Bmatrix} \tag{1}$$

where t is the time. $M=M(\theta_1,\theta_2,u,u^2,t)$ is a 3 × 3 mass moment of inertia matrix. K is a 3 × 3 stiffness matrix. C is a 3 × 3 damping matrix. $G_1=G_1(\theta_1,\theta_2,u,t),G_2=G_2(\theta_1,\theta_2,u,t)$, and $G_3=G_3(\theta_1,\theta_2,t)$ are a non-linear displacement-dependent equations resulting from the gravitational potential energy of the system. $B=B(\theta_1,\theta_2,u,\dot{\theta}_1,\dot{\theta}_2,\dot{u},t)$ is a 3 × 3 non-linear velocity-dependent matrix resulting from the kinetic energy of the system. F is a 3 × 1 active moment vector of the muscle. $\mathcal{T}$ is a 3 × 1 voluntary torque vector exerted by the muscles of the patient to place his upper limbs in a specific postural position θ_p. The restoring force function $s(u)$ of the NES is :

$$s(u)=\begin{cases}k_N(u+d) & if\ u<-d\\ 0 & if\ -d\leq u\leq d\\ k_N(u-d) & if u>d\end{cases} \tag{2}$$

where, k_N is the stiffness, and 2d is the clearance of the non-smooth NES. The viscous damping c_N of the NES is assumed to be linear.

2.2 Partially linearized equations

Equations of motion referred to the principle system without NES can be linearized using Taylor series multivariable linearization method [17], by keeping a good agreement with the corresponding non-linear equations [9]. The non-linearity of the NES in the global system is preserved by partially linearizing the equations of motion in (1). The angular displacements θ_1 and θ_2 of the shoulder and elbow joints are linearized around the postural positions θ_{p_1} and θ_{p_2}, respectively, by keeping u terms unchanged. The equation of motion of the partially linearized system of (1) can be written in such a way, so that the last equation of motion (corresponding to the NES) is separated from the first two equations as follows :

$$\begin{cases}M_A\begin{Bmatrix}\Delta\ddot{\theta}_1\\ \Delta\ddot{\theta}_2\end{Bmatrix}+M_V\Delta\ddot{u}+K_A\begin{Bmatrix}\Delta\theta_1\\ \Delta\theta_2\end{Bmatrix}+K_V\Delta u+C_A\begin{Bmatrix}\Delta\dot{\theta}_1\\ \Delta\dot{\theta}_2\end{Bmatrix}+C_V\Delta\dot{u}=\begin{Bmatrix}\Delta F_1\\ \Delta F_2\end{Bmatrix}\\ M_H\begin{Bmatrix}\Delta\ddot{\theta}_1\\ \Delta\ddot{\theta}_2\\ \Delta\ddot{u}\end{Bmatrix}+K_H\begin{Bmatrix}\Delta\theta_1\\ \Delta\theta_2\\ 0\end{Bmatrix}+C_H\begin{Bmatrix}\Delta\dot{\theta}_1\\ \Delta\dot{\theta}_2\\ \Delta\dot{u}\end{Bmatrix}+s(\Delta u+u_{0_p})-s(u_{0_p}))=0\end{cases} \tag{3}$$

where $\Delta\theta=\theta-\theta_p$ and $\Delta u=u-u_{0_p}$, similarly for the velocity and acceleration terms. $\Delta F=F(t)-F(t=0)$. The value u_{0_p} corresponds to the initial displacement of the NES due to postural position, for to the applied $\mathcal{T}$. $M_A=M_A(\theta_{p_1},\theta_{p_2},u_{0_p},u_{0_p}^2,\Delta u,(\Delta u)^2,t)$, $M_V=M_V(\theta_{p_1},\theta_{p_2},\Delta\theta_2)$, $M_H=M_H(\theta_{p_1},\theta_{p_2})$, $K_A=K_A(\theta_{p_1},\theta_{p_2},u_{0_p},\Delta\theta_1,\Delta\theta_2,\Delta u)$, $K_V=K_V(\theta_{p_1},\theta_{p_2},\Delta u)$, $K_H=K_H(\theta_{p_1},\theta_{p_2})$, and $C_A=C_A(\theta_{p_1},\theta_{p_2},u_{0_p},\Delta u,\Delta\dot{u})$, where M_A, K_A, and C_A are 2 × 2 matrices, M_V and K_V are 2 × 1 vectors, and M_H, K_H, and C_H are 1 × 3 vectors.

The partially linearized equation of variables $(\Delta\theta_1, \Delta\theta_2, \Delta u)$ in (3), is transformed to the modal coordinates of variables (W_1, W_2, U), using the transformation equation :

$$\begin{Bmatrix} \Delta\theta_1 \\ \Delta\theta_2 \end{Bmatrix} = P \begin{Bmatrix} W_1 \\ W_2 \end{Bmatrix} \quad and \quad \Delta u = U, \quad such\, that \;\; P = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} \tag{4}$$

where P is a matrix of columns representing the eigenvectors V_i of the principle system. Some modal matrices are defined in the modal coordinates depending on the equations of motion of the principle system :

$$P^{-1} M_0^{-1} \Delta F = \varepsilon \Delta f \tag{5}$$

$$P^{-1} M_0^{-1} K_0 P = \begin{bmatrix} \omega_1^2 & 0 \\ 0 & \omega_2^2 \end{bmatrix} \tag{6}$$

$$P^{-1} M_0^{-1} C_0 P = \varepsilon \begin{bmatrix} \lambda_{p_1} & 0 \\ 0 & \lambda_{p_2} \end{bmatrix} \tag{7}$$

where M_0, K_0, and C_0 are the mass moment of inertia, stiffness, and damping matrices of the linearized principle system, knowing that ω_1 and ω_2 represents its eigenfrequencies. ε is defined to be $\frac{m_N}{m_1}$. Other variables depending on parameters of NES, which can be defined are :

$$P^{-1} M_0^{-1} s(U) = \varepsilon \Lambda(U) \tag{8}$$

$$\widetilde{k}_N = \frac{k_N}{m_N} \;\; and \;\; \frac{c_N}{m_1} = \varepsilon \lambda_N \tag{9}$$

2.3 Complexification of the system

The complex variables of Manevitch (without pre-stressing terms) [18], are introduced to the partially linearized equations in the modal coordinates :

$$\begin{cases} \varphi_1 e^{i\omega t} = \dot{W}_1 + i\omega W_1 \\ \varphi_2 e^{i\omega t} = \dot{W}_2 + i\omega W_2 \\ \psi e^{i\omega t} = \dot{U}_N + i\omega U_N \end{cases} \tag{10}$$

where $\omega = \omega_1 - \sigma_1\varepsilon$ in the first equation, and $\omega = \omega_2 - \sigma_2\varepsilon$ in the second one. ω of the last equation depends on the studies case, where resonance is assumed to occurs at the first eigenfrequency of the system. $\varphi_1 = N_1 e^{i\delta_1}$, $\varphi_2 = N_2 e^{i\delta_2}$, and $\psi = N_u e^{i\delta_u}$, such that N is the amplitude and δ is the phase. The multiple scale method [19] is used to treat the system at different timescales, slow timescale $\tau_0 = t$ and fast timescale $\tau_j = \varepsilon^j t$, such that $j = \{1, 2\}$ [20]. Different timescales are coupled using the physical parameter ε. A Galerkin method or a truncated Fourier series (constant terms and first harmonics) [20] is applied, to the equation of motion transformed to the modal coordinates, in order to obtain the averaged equation.

3 Results

The averaged equation is studies at different timescales. This means that we should study the system equations at different orders of ε. In the current work, the behaviour of the system is presented for the fast time scale (ε^0 order) of the system equations. The slow invariant manifold (SIM) is obtained as :

$$\begin{gathered} \frac{i}{2} a_1 \varphi_1 + \frac{i}{2} a_2 \varphi_2 = \frac{i(\omega_1^2 - G(|\psi|^2)) + \lambda_N \omega_1}{2\omega_1} \psi, \quad \text{such that} \\ a_1 = \left(\frac{-g \sin(\theta_{p_1} + \theta_{p_2}) - (d_N + l_1 \cos(\theta_{p_2}))\omega_1^2}{\omega_1} \right) V_{11} + \left(\frac{-g \sin(\theta_{p_1} + \theta_{p_2}) - d_N \omega_1^2}{\omega_1} \right) V_{21}, \;\; \text{and} \\ a_2 = \left(\frac{-g \sin(\theta_{p_1} + \theta_{p_2}) - (d_N + l_1 \cos(\theta_{p_2}))\omega_2^2}{\omega_2} \right) V_{12} + \left(\frac{-g \sin(\theta_{p_1} + \theta_{p_2}) - d_N \omega_2^2}{\omega_2} \right) V_{22} \end{gathered} \tag{11}$$

where g is the gravitational acceleration of the system. $G(|\psi|^2))$ is the averaged form of $s(u)$ in (2) [21]. The SIM obtained from the analytical study is compared to the numerical response obtained from direct numerical integration of the equations (3) and (4) using ode45 in Matlab.

Analytic and numerical equations are solved for the system operating at $\theta_{p_1} = -90°$ and $\theta_{p_2} = 0°$, which corresponds to $u_{0_p} = 0$ m. Initial conditions for the amplitude of the assigned variables of Manevitch are $N_{0_1} = 1$, $N_{0_2} = 1.57$, and $N_{0_u} = 0.0352$, which corresponds to the shoulder joint, elbow joint, and NES, respectively. These initial conditions corresponds to $\theta_{0_1} = -108.9°$, $\theta_{0_2} = 22.7°$, and $u_0 = 5.3$ mm, representing the initial angular displacements at the shoulder joint and elbow joint, and the displacement of the NES in the physical coordinates. The transformation back from modal to physical coordinates is obtained using (4). Parameters of the coupled system used for simulations are represented in 2, in addition to the physical parameters of the principle system found in Table 1.

TABLE 2 – Coupled system parameters

ε	λ_{p_1}	λ_{p_2}	λ_N	$\tilde{k}_N$	2d	$\delta_2 - \delta_1$
10^{-3}	1	1	0.5	90	3×10^{-2}	$-\Pi$

The response of the system in the physical coordinates obtained numerically before and after the addition of the NES, is shown in Figure 2. Results are shown at the beginning and end of the simulation. Although the chosen ε is very small, corresponding to a NES of mass $m_N = 2.1\ g$, the NES is efficient in reducing the amplitude of oscillation after a period of time. The tested NES, of a mass ratio 0.055 % (calculated relative to the total mass of the upper limb), was able to reduce the flexion-extension angular displacement amplitude of the shoulder and elbow joints. Note that higher reduction with faster respond can be obtained at the beginning of the simulation for a higher chosen value of ε.

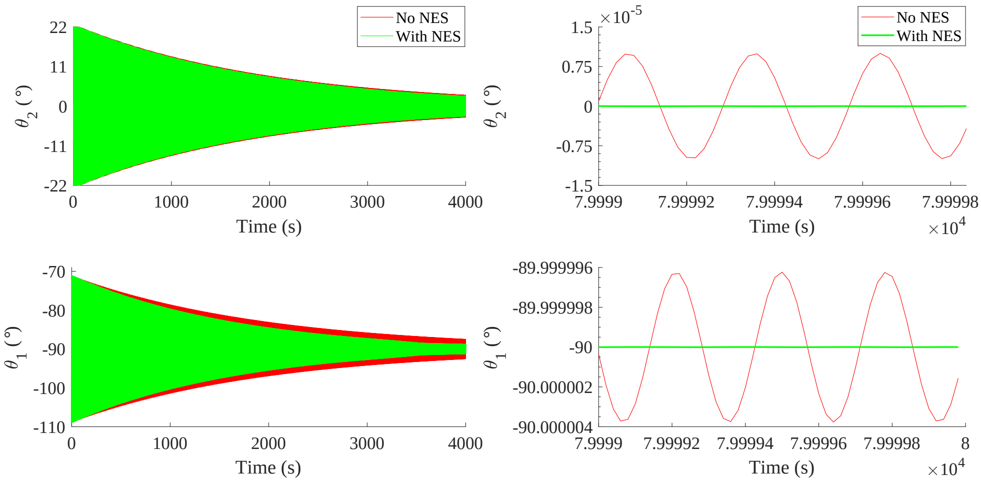

FIGURE 2 – Angular displacement signals at the shoulder and elbow joints simulated numerically with and without NES

A comparison between numerical results and SIM is shown in Figure 3, where the initial and final position of the simulation are indicated. Results shows that the numerical result is following the SIM, which validates our results. Further analysis should be done for the slow timescale to study the equilibrium and singular points of the system.

4 Conclusion

A non-smooth non-linear energy sink (piece-wise linear) shows its ability to reduce the angular displacement amplitude at the joints of the upper limbs. This non-linear passive controller is able to operate for a very small mass added to the arm. Measurements for the tremor signals of patients with

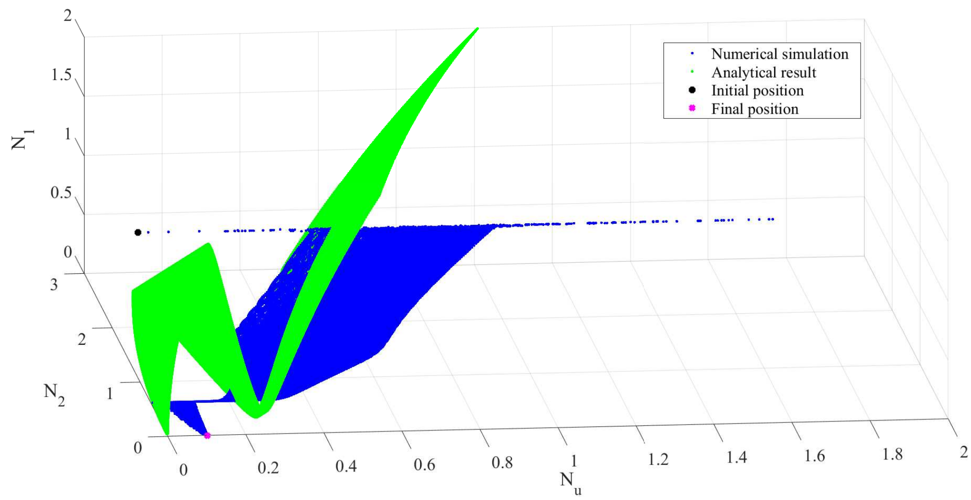

FIGURE 3 – Numerical results compared to the SIM

essential tremor will be provided to support this study. The controller will be designed in the form of a wearable bracelet before it is tested on the arm of a real patient.

Acknowledgement

The authors would like to thank CNRS for supporting this work in the framework of the "programme de prématuration du CNRS".

Références

[1] G. Deuschl, P. Bain, M. Brin. *Consensus statement of the Movement Disorder Society on tremor*, Movement Disorders, 13(S3), 2-23, 1998.

[2] S. Morrison, G. Kerr, P. Silburn. *Bilateral tremor relations in Parkinson's disease : Effects of mechanical coupling and medication*, Parkinsonism and Related Disorders, 14(4), 298-308, 2008.

[3] S. Smaga. *Tremor-Problem-Oriented Diagnosis*, American Family Physician, 68, 1545-1552, 2003.

[4] S. Reich. *Common disorders of movement : Tremor and Parkinson's disease*, In Principles of Ambulatory Medicine, Williams and Wilkins : Baltimore, MA, USA, 1217-1229, 1995.

[5] M. J. Rosen. *Multiple degree of freedom damped hand controls*, US Patent 5,107,080, 1992.

[6] N. Stone, K. Kaiser, R. D. White. *Autotuning of a PID controller for an active vibration suppression device for the treatment of essential tremor*, In ASME 2006 international mechanical engineering congress and exposition, Chigaco, Illinois, USA, 855-861, 2006.

[7] S. M. Hashemi, M. F. Golnaraghi, A. E. Patla. *Tuned vibration absorber for suppression of rest tremor in Parkinson's disease*, Medical and Biological Engineering and Computing, 42(1), 61-70, 2004.

[8] S. Gebai, M. Hammoud, G. Cumunel, G. Foret, E. Roze, E. Hainque. *Experimental Testing of Passive Linear TMD for Postural Tremor Attenuation*, Applied Sciences, 11(21), 9809-9821, 2021.

[9] S. Gebai. *Optimization of Passive Cantilever-Type Tuned Mass Damper to Reduce the Hand Postural Tremor*, Ph.D. Thesis, Université Paris-Est, Paris, France, 8 July 2020.

[10] A. True Savadkoohi, C.-H. Lamarque, C. Goossaert. *Nonlinear passive tremor control of human arm*, Mechanical Systems and Signal Processing, 146, 107041-107061, 2021.

[11] A. Vakakis, O. Gendelman, L. Bergman, D. McFarland, G. Kerschen, Y. Lee. *Nonlinear Targeted Energy Transfer in Mechanical and Structural Systems*, Solid Mechanics and Its Applications, Springer, Netherlands, 2008.

[12] J. M. Dolan, M. B. Friedman, M. L. *Nagurka. Dynamic and loaded impedance components in the maintenance of human arm posture*, IEEE transactions on systems, man, and cybernetics, 23(3), 698-709, 1993.

[13] K. M. Jackson, J. T. Joseph, S. J. Wyard. *A mathematical model of arm swing during human locomotion*, Journal of Biomechanics, 11(6), 277-289, 1978.

[14] E. Harless. *The static moments of human limbs*, Treatise of the Mathematical Physical Class of the Royal Bavarian Academy of Sciences, 8, 69-96, 1860.

[15] G. Hurel, A. True Savadkoohi, C.-H. Lamarque. *Passive control of a two degrees-of-freedom pendulum by a non-smooth absorber*, Nonlinear Dynamics, 98(4), 3025-3036, 2019.

[16] D. J. Inman. *Engineering Vibrations*, Upper Saddle River, NJ : Virginia Tech, Prentice Hall, 2008.

[17] A. Babiarz, R. Bieda, K. Jaskot, J. Klamka. *The dynamics of the human arm with an observer for the capture of body motion parameters*, Bulletin of the Polish Academy of Sciences, Technical Sciences, 61(4), 955-971, 2013.

[18] L. I. Manevitch. *The description of localized normal modes in a chain of nonlinear coupled oscillators using complex variables*, Nonlinear Dynamics, 25(1), 95-109, 2001.

[19] A. H. Nayfeh, D.T. Mook. *Nonlinear oscillations*, Wiley, New York, 1979.

[20] A. True Savadkoohi, C.-H. Lamarque, M. Weiss, B. Vaurigaud, S. Charlemagne. *Analysis of the 1 : 1 resonant energy exchanges between coupled oscillators with rheologies*, Nonlinear Dynamics, 86(4), 2145-2159, 2016.

[21] O. V. Gendelman. *Targeted energy transfer in systems with non-polynomial nonlinearity*, Journal of Sound and Vibration, 315(3), 732-745, 2008.