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Initial conditions to account for moving loads in truncated domains and propagating waves generated in the sub-Rayleigh regime by the quasi-static excitation of a train passing on a heterogeneous domain

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Résumé — When using FEM to solve wave propagation problems, the introduction of moving loads generates propagating parasitic waves due to the truncation of the computational domain. These waves can be avoided by introducing initial conditions obtained from a preliminary simulation. This technique is applied in the context of rail transport. A static load moving in sub-Rayleigh regime on an invariant by translation medium generates evanescent waves and is not taken into account in the prediction of ground vibrations in the far field. However, when the medium is heterogeneous propagating waves are generated.
Mots clés — Moving Load, Wave Propagation, Railway Vibration, Finite Element Method.

1 Introduction

Moving loads are commonly used in engineering to model for instance the passage of a train. Analytical solutions exist in the case of a load moving in an unbounded domain [1] or at the surface of a half-space [2]. In the case when the geometry and parameters are invariant in the direction along which the load moves, the problem can be solved easily in the frequency domain after a change of space-time variables. When the geometry and material parameters are periodic in the direction along which the load moves, a Bloch-Floquet analysis allows to solve the problem numerically only on the (small) periodicity cell. In all other cases, among which many of interest for the industrial applications, the problem has to be solved in the time domain, over a domain truncated for instance with Perfectly Matched Layers (or other such absorbing boundary conditions or layers). The introduction of a moving load in such a configuration requires a specific treatment that will be discussed in the first part of this presentation.

The case of a moving load at the surface of a homogeneous half space is considered. The truncation of the computational domain induces parasitic waves both when the loads enters the domain and when it leaves it [3]. This is shown numerically using a parallel High Performance Computing software based on the spectral element method [4]. A solution based on the introduction of pre-computed initial conditions is presented in section 2. Displacements and velocities are saved in a first simulation and reused in a second one as initial conditions. The same initial conditions can be reused in different numerical simulations. This is illustrated in the second part of the presentation on a series of applications of interest for the railway industry.

The excitation induced by a moving train can be separated into a quasi-static and a dynamic component [5]. For the prediction of ground-borne vibration in the far-field only the latter is usually considered. However, in several railway applications, an underestimation of the vibrations is often observed when comparing to measurements. The Section 3 deals with the far-field contribution of the quasi-static loading of a train passing at sub-Rayleigh speed on a heterogeneous domain. Two cases where the translation-invariance of the medium is lost are investigated. First a transition from a ballasted track to a slab track is considered. Then, a track model accounting for the ballast heterogeneity is considered using a randomly-fluctuating heterogeneous continuum model [6]. In both cases the same initial conditions are used to avoid the presence of parasitic waves.

2 Parasitic waves generated by the introduction of a moving load over a bounded domain in a time domain simulation

Let $\mathbf{x} = (x, y, z)$ be a generic position in a Cartesian coordinate system. The displacement field in a domain Ω can be obtained by solving the equation of motion :

$$\nabla \cdot \sigma(\mathbf{x},t) + \mathbf{f}(\mathbf{x},t) = \rho \frac{\partial^2 \mathbf{u}(\mathbf{x},t)}{\partial t^2}, \tag{1}$$

where $\mathbf{f}(\mathbf{x},t)$ is bulk force, $\mathbf{u}(\mathbf{x},t) = (u, v, w)$ is the displacement vector, σ is the stress tensor and $\rho(\mathbf{x})$ is the density. The domain Ω is assumed to be a half-space, so that the computational approximation of Eq. (1) by traditional Finite Element Methods is completed with absorbing boundary layers [7, 8, 9, 10]. A Neumann boundary condition is considered at the surface of the half-space :

$$\sigma \cdot \mathbf{n} = \mathbf{g}(\mathbf{x},t), \quad \mathbf{x} \in \partial\Omega. \tag{2}$$

Finally, the equilibrium Eq. (1) and boundary conditions are completed with homogeneous initial conditions :

$$\mathbf{u}(\mathbf{x}, t=0) = \mathbf{0}, \quad \frac{\partial \mathbf{u}}{\partial t}(\mathbf{x}, t=0) = \mathbf{0} \tag{3}$$

A concentrated normal load, moving with a uniform velocity V = 90 m/s over a set of equally-spaced points d = 0.1 m along a line $(y = 0, z = 0)$ at the surface of a half-space bounded by $z = 0$ is considered (see Fig. 1). A discrete load model is considered because it is close to the real situation in many engineering applications. This is for instance the case of railway applications, where loads are transmitted from the train/track system to the ballast through sleepers, separated by a constant distance. We assume

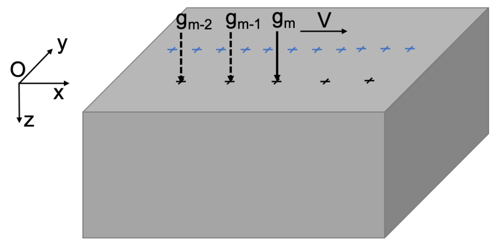

FIGURE 1 – A constant force moving at the surface of a homogeneous half-space and applied at a discrete set of points along a line. Sensors are located along a line parallel to the load line (blue crosses).

that the half-space is elastic, isotropic and homogeneous with velocities C_P = 600 m/s, C_S = 450 m/s, C_R = 380 m/s and density ρ = 1800 kg/m^3. The Neumann boundary condition (Eq. (2) with $\mathbf{n} = -\mathbf{e}_z$) at the surface of a half-space is constituted of a sum of traction forces centered on positions $x = md$ and times $t - md/V$, $m \in \mathbb{Z}$:

$$\forall m \in \mathbb{Z}, \quad \mathbf{g}_m(\mathbf{x},t) = F_0 \left(t - \frac{md}{V} \right) \delta(x - md)\delta(y)\mathbf{e}_z, \tag{4}$$

such that

$$\mathbf{g}(\mathbf{x},t) = \sum_{m \in \mathbb{Z}} \mathbf{g}_m(\mathbf{x},t). \tag{5}$$

The load $F_0(t)$ considered is an impulse applied on the ground and containing frequencies in the band 10-100Hz. The time representation of the load and it frequency content are presented in Fig.2.

The time-based solution of wave propagation induced by the moving load is performed with a spectral element-based [4, 11] software developed by CentraleSupélec, CEA (French Atomic Energy Commission), IPGP (Paris Institute of Earth Physics) and CNRS (The French National Centre for Scientific Research). The displacements induced in the soil at sensors along a line parallel to the loading line and at 2 m from it are presented in Fig. 2 (Left). The quasi-static displacements induced by the moving load but also two additional waves propagating in the soil at the Rayleigh wave velocity and generated at the

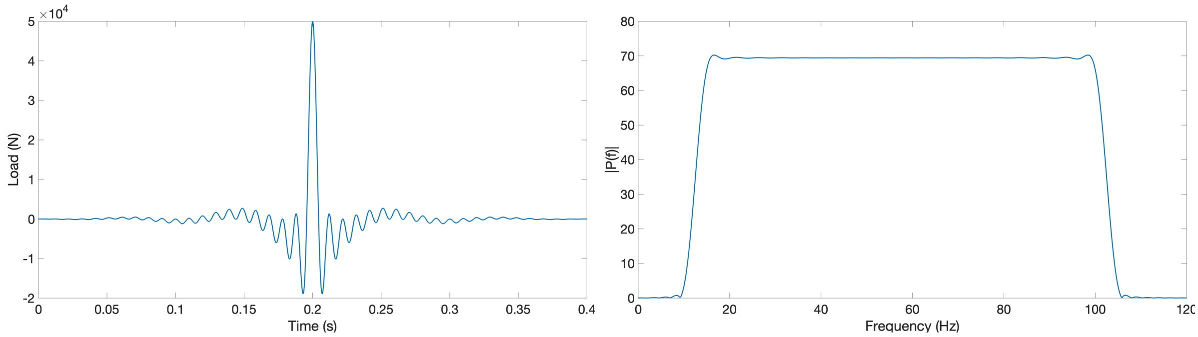

FIGURE 2 – Time history (left) and spectrum (right), of the excitation $F_0(t)$.

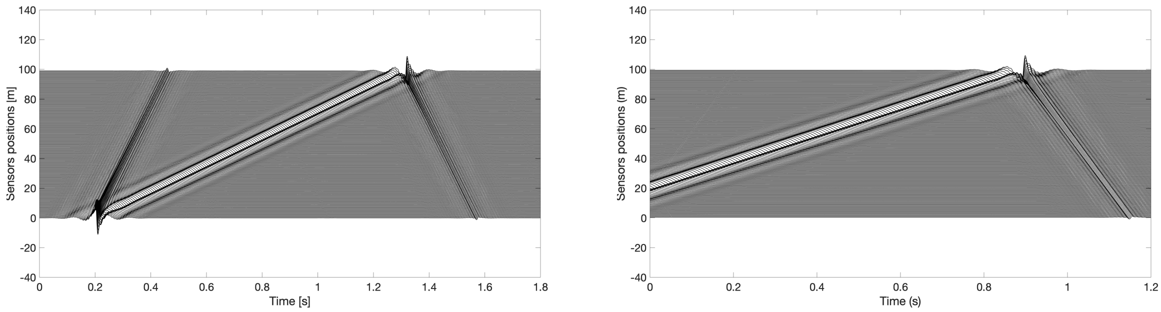

FIGURE 3 – Vertical displacements induced by a moving load, computed using a SEM solver with (Left) homogeneous initial conditions and (Right) heterogeneous initial conditions, at sensors located along a line parallel to and 2 m away from the loading line.

boundaries of the loading support are identified. What happens is that the quasi-static appearance of the displacement induced by the unbounded load actually arises as a complex interference between waves coming from the different sources. In the middle of the loading line, this interference can take place normally. However, on the boundaries of the loading line, the sources outside the computational domain are lacking. The absence of the corresponding waves means that full interference cannot take place. The waves emitted from the first and last few sources are therefore not properly compensated and reappear as propagating waves in the soil.

To solve the problem, the solution proposed is to replace the homogeneous initial conditions by non-homogeneous ones :

$$\mathbf{u}(\mathbf{x}, t=0) = \mathbf{u}_0(\mathbf{x}), \quad \frac{\partial \mathbf{u}}{\partial t}(\mathbf{x}, t=0) = \mathbf{v}_0(\mathbf{x}) \tag{6}$$

These non homogeneous initial conditions are estimated numerically. A preliminary simulation is performed for which the displacement and velocity fields are saved in a box at some final time. The box is chosen large enough so that the fields are sufficiently close to zero at the boundaries. The final time at which data are saved is chosen so that the area of interest defined by the box is no longer polluted by interference waves. The saved displacement and velocity fields are then injected into the simulation of interest as initial conditions. The displacement induced in the soil when non-homogeneous displacement and velocity fields are used as initial conditions is presented in Fig. 2 (Right). The initial parasitic wave due to the truncation of the computational domain is completely suppressed.

3 Far-field contribution of the quasi-static loading of a train passing at sub-Rayleigh speed on a heterogeneous domain

Two railway applications of moving load on heterogeneous medium are presented. For both cases, a simplified model of the track is considered where only the ballast and the soil are modeled. The technique presented in the previous part is applied to obtain the initial conditions that are then used for both simulations.

In the preliminary simulation, the soil is assumed linear, elastic and homogeneous with pressure wave velocity C_P = 600 m/s, shear wave velocity C_S = 350 m/s and density ρ = 1800 kg/m^3. The ballast is also linear, elastic and homogeneous with C_P = 397 m/s, C_S = 212 m/s and ρ = 1700 kg/m^3. The 3D model for the computation of the initial conditions is presented in Fig. 4 (left). The load considered is that induced by a bogie on a typical railway track, estimated with the approach of [12] and presented in Fig. 5.

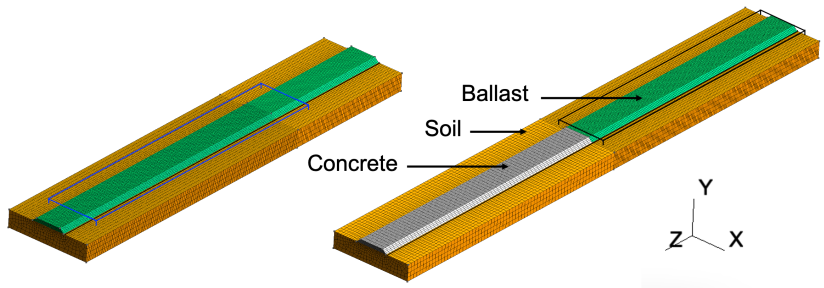

FIGURE 4 – 3D models of railway track : ballasted railway track for the computation of initial conditions (left), and railway track with a transition from a ballasted track to a slab track (right). The boxes indicate where the initial conditions were obtained (blue box) and where they are introduce in the simulation of interest (black box). Sensors are located along a line (black line) parallel to and 10 cm away from the ballast. Perfectly Matched Layers are not represented.

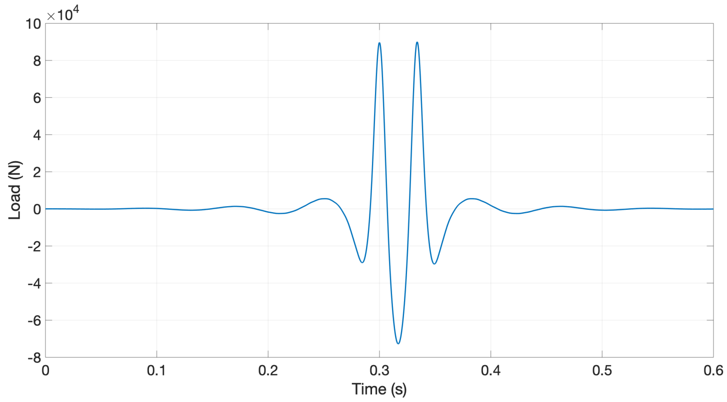

FIGURE 5 – Time dependence of the bogie load

The first application is the passage of a bogie through a transition between a ballasted track and a slab track (see Fig 4 (right)). The black box in Fig 4 (right) indicates the area where the initial conditions are introduced. The soil and the ballast layer are assumed homogeneous with the same properties as in the preliminary simulation. The concrete is also assumed homogeneous with pressure wave velocity C_P = 3725 m/s, shear wave velocity C_S = 2236 m/s and density ρ = 2400 kg/m^3. The displacement induced at the bottom of the track are presented in Fig. 6. Evanescent waves are generated when the load is moving on a homogeneous material. However, propagating waves are generated when the load passes through the transition zone. The position of the transition area is represented by the green line.

In the second application a ballasted track is considered and a randomly-fluctuating heterogeneous continuum model is used to model the ballast layer [6]. The average values of the ballast pressure and shear wave velocities and the ballast density are those of the homogeneous case. The coefficient of variation of these parameters is equal to 2. The displacement field induced by the moving load are represented in Fig. 7. Propagating waves are present in the whole domain. A space and time shifting is performed for each sensor as follow :

$$u_n^*(t) = u\left(t - \frac{nD}{v}\right) \tag{7}$$

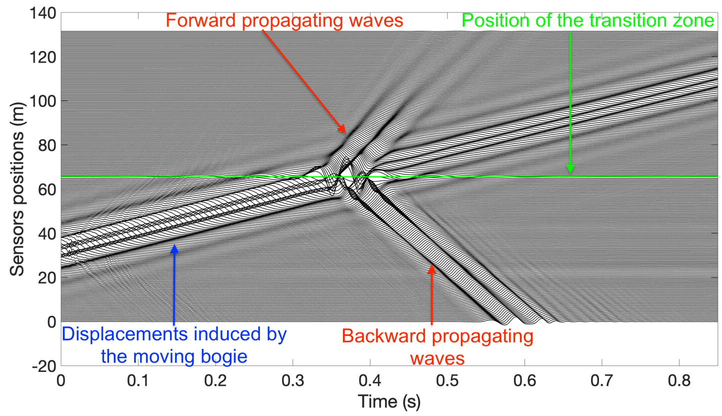

FIGURE 6 – Vertical displacements induced in the soil at the footprint of the ballast layer near a transition area between a ballasted track and a slab track

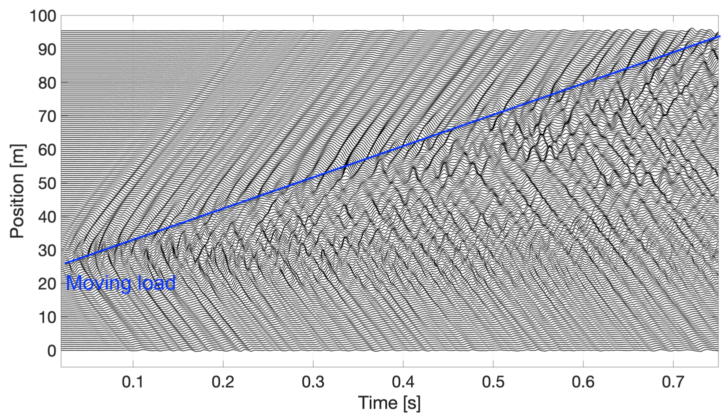

FIGURE 7 – Vertical displacements induced in the soil at 4 m from the ballast layer with a heterogeneous model of the ballast

where n is the number of the sensor and D the spacing between two consecutive sensors. The displacement field u^* is analyzed as statistically homogeneous in space. Therefore each sensor is considered as one realization of a random process. The Fig. 8 presents the Fourier transform of displacements for the homogeneous and heterogeneous case. The average of the Fourier transforms of the heterogeneous case, calculated from results on 70 sensors, is similar to that of the homogeneous case although the frequency content for one realization is very different. The heterogeneities lead to contributions in the [30-110] Hz frequency range which is well above the frequency content of the quasi-static excitation considered here [10-40] Hz.

4 Conclusion

The introduction of initial conditions in a time domain simulation allows to avoid the propagation of parasitic waves due to the truncation of the physical domain when problems with moving loads are considered. This technique allows to reduce the computation costs by reducing the size of the domain and the computation time. It is used in this paper for railway applications where it is necessary to model a moving train.

A load moving at a sub-Rayleigh speed on an unbounded and invariant by translation medium generates an evanescent wave front located in the vicinity of the load. It is therefore not taken into account in the prediction of far field vibrations. However this study shows, for railway applications, that when a train is moving on heterogeneous domain the contribution of the quasi-static load on the far field vibration has to be taken into account due to generated propagating waves. When using a heterogeneous model for ballast layer, the heterogeneities lead to important displacements in a frequency range higher

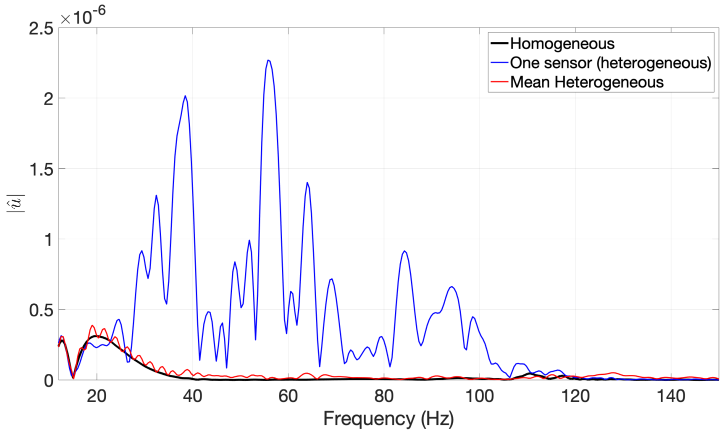

FIGURE 8 – Comparison of the Fourier transforms of displacement for the homogeneous and the heterogeneous case

than the frequency content of the quasi-static excitation.

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