CSMA 2022
15ème Colloque National en Calcul des Structures
16-20 mai 2022, Presqu'île de Giens (Var)

Robustness evaluation strategy for optimized composite lightweight structures

C. Becker[1,2], P. Lardeur[1], P. Nicolay[3], F. Druesne[1]

[1] *Université de technologie de Compiègne, CNRS, Roberval (Mécanique, énergie et électricité), Centre de recherche Royallieu - CS 60 319 - 60 203 Compiègne Cedex, France, {clarissa.becker}@utc.fr*

[2] *ADMiRE Lab - Additive Manufacturing, intelligent Robotics, Sensors and Engineering, School of Engineering and IT, Carinthia University of Applied Sciences, Europastrasse 4, 9524 Villach, Austria*

[3] *Carinthia Institute for Smart Materials (CiSMAT), Carinthia University of Applied Sciences, Europastasse. 4, 9524 Villach, Austria*

Résumé — Technological innovations such as Continuous Filament Fabrication enrich the manufacturing possibilities for lightweight structures, essentially because they enable more complex geometries. However, design optimization is more challenging and requires the increasing use of numerical optimization methods. Continuous Filament Fabrication is a source of various uncertainties. Therefore, the high robustness of the mechanical behavior of optimized designs is desirable. Two approaches to obtain robust designs are presented. A stochastic method, the Certain Generalized Stresses Method, is used to study the robustness of structures.

Mots clefs — Additive manufacturing, Truss structures, Uncertainties, CGSM.

1. Introduction

In recent years, additive manufacturing has impacted the design of lightweight structures [1]. With it, much more complex components can be manufactured than with traditional manufacturing processes [2,3]. For lightweight applications, the use of continuous composite materials is attractive due to their excellent material properties (stiffness, strength) [4,5]. The most used additive manufacturing technology for continuous composite materials is Continuous Filament Fabrication [6,7]. However, new challenges are imposed by the Continuous Filament Fabrication process [8]. On the one hand, numerical optimization tools become essential in the design process to benefit from the high geometrical freedom in fabrication [9,10]. On the other hand, various sources of uncertainty in material properties and manufacturing are associated with this process (e.g., in [11,12]). As a result, variability in material and physical parameters can be observed in manufactured parts, which can influence the static or dynamic structural response. Therefore, creating robust designs with a low sensitivity to variability of material and physical parameters is desirable.

In this work, the Certain Generalized Stresses Method (CGSM) is used to study the influence of parameter variability on the robustness of the structure. The stochastic method is computationally efficient due to a reduced number of finite element analyses and can be used for a large number of uncertain parameters. For the implementation of the robustness evaluation in the design process, two approaches are proposed: first, the uncoupled approach, which is a two-step robust selection method, and second, the coupled approach in the form of robust optimization.

This paper continues as follows: the two approaches to consider robustness evaluation in the design process are proposed in Section 2. Then, a description of the stochastic method CGSM, including the CGSM formulation for bar trusses and the implementation in the uncoupled approach follows in Section 3. The implementation is then demonstrated with a classical MBB (Messerschmitt-Bölkow-Blohm) beam example in Section 4. Conclusions and future work are given in Section 5.

2. Robustness evaluation strategy in the design process

A structure is robust if its static or dynamic structural response is not sensitive to variability in material and physical parameters. The variability in material or physical properties, load direction or magnitude, boundary conditions, and geometry means that these parameters are not deterministic. As a result, the quantities describing the structural behavior under load, such as displacement, are not deterministic either. Stochastic modeling can be used to study the influence of variability on the structural response. With a probabilistic approach, the stochastic model determines how likely a particular structural response is expected.

2.1. An uncoupled approach

The robustness of structures can be achieved in several ways. One approach is like the concept of generative design [13]. In generative design, different possible designs which meet overall design criteria (e.g., minimum weight or maximum stiffness) are created. In an iterative process, the designs are gradually improved through selection by either a human interaction or an algorithm. The uncoupled approach can be implemented in a two-step process where, first, optimized structures which meet the overall design criteria are created. Second, the robustness of these structures is evaluated, taking into account parameter variability. This procedure is an uncoupled approach, where an optimized and robust design is obtained in two steps using a selection method.

There are some advantages to a selection method. The two different steps of optimization and robustness selection are independent from each other. Thus, a particular robustness selection method can be used in combination with any optimization algorithm. Moreover, if the optimization objective changes, for instance, from weight minimization to stiffness maximization, it does not influence the robustness selection method. However, there are also disadvantages to the uncoupled approach. The use of an iterative process to obtain optimized and robust designs may be time-consuming. Furthermore, the globally optimal and most robust design may not be found.

2.2. A coupled approach

Robust design optimization is a method to consider parameter variability within the optimization algorithm. There are multiple possibilities to consider the design robustness in an optimization formulation, for instance, as an optimization constraint or optimization objective. In the first case, the optimization objective remains unchanged to maximize the stiffness or minimize the weight of the structure. The design robustness is implemented as a constraint, limiting the accepted variability of structural performance, for instance, displacement variability. In the second case, a multi-objective optimization is targeted, where two different optimization objectives are considered [14]. This is, for instance, to minimize the nominal displacement and the displacement variability simultaneously.

A method which considers robustness within the optimization step is a coupled approach. In contrast to the uncoupled approach, a globally optimal and robust solution is more likely to be found. However, in the coupled approach, the evaluation of robustness is dependent on the optimization algorithm. Depending on the optimization algorithm and the implementation of robustness in it, convergence can be affected. Furthermore, two optimization objectives may be contradictory in a multi-objective formulation. Such behavior can be studied with a weighted objective function and mapped in a Pareto front. Finally, the coupled approach is more difficult to develop than the uncoupled approach, but it is also more promising.

3. The Certain Generalized Stresses Method (CGSM)

3.1. Principle

The principle of the stochastic method CGSM is to build a metamodel based on a finite element model. The metamodel allows to define physical and material properties, which can be considered as uncertain parameters. The propagation of uncertainty is performed with a Monte Carlo simulation, where no further finite element analyses are needed. The CGSM assumes the independence of the generalized forces in the structure from the uncertain parameters. This assumption is exact for statically determinate structures so that, in this case, the method leads to exact calculations of the quantities of interest (e.g., displacement). However, the CGSM is also applicable to statically indeterminate structures, for which it leads to an approximation. As proposed by Yin et al. [15], an error indicator is useful to estimate the quality of the mean value or the standard deviation of the quantity of interest. For a low number of trials, the results obtained by the CGSM are compared to the results of a direct Monte Carlo simulation, using the same values for the uncertain parameters.

3.2. CGSM formulation for bar trusses

The CGSM is used to analyze thin-walled structures, bars and beam trusses, for which different formulations exist. The following formulation for bar trusses is based on Lardeur et al. [16]. The objective is to observe the variability of displacement U depending on the elasticity modulus E as an uncertain parameter. A given truss structure is modeled with n bar elements. Further assuming a constant axial force throughout an element, the global strain energy π_{int} for a truss with n elements is:

$$\pi_{int} = \frac{1}{2}\sum_{i=1}^{n} \frac{N_i^2 l_i}{A_i E_i} \tag{1}$$

where N_i is the axial force in the element i and further, l_i is the length, A_i the cross-sectional area and E_i the elasticity modulus of the element i. The Castigliano theorem states:

$$U = \frac{\partial \pi_{int}}{\partial F} \tag{2}$$

from which the displacement U at a point P in the direction of interest can be obtained. F is a physical or fictive force in the direction of interest. To utilize equation (2), the axial force N_i is decomposed into:

$$N_i = N_i' + F N_i'' \tag{3}$$

where N_i' is the axial force in element i due to all external forces except at point P in the direction of interest and $N_i''$is the axial force in element i due to a unitary force applied at point P in the direction of interest. Based on the CGSM assumption, the axial forces N_i' and N_i'' are independent from the uncertain parameters. Substituting equations (1) and (3) into equation (2) leads to the displacement:

$$U = \sum_{i=1}^{n} \frac{N_i'' l_i (N_i' + F N_i'')}{A_i E_i} \tag{4}$$

where l_i, N_i', N_i'', F and A_i are deterministic values and E_i is the uncertain parameter of element i. If the force F is fictive and not physically present, $F = 0$ is applied in equation (4).

The coefficient of variation c.o.v. is used to evaluate the variability of the displacement and is defined as:

$$\text{c.o.v.}(U) = \frac{\sigma(U)}{m(U)} \quad \textbf{(5)}$$

where $m(U)$ is the mean value and $\sigma(U)$ is the standard deviation of the displacement.

3.3. Implementation of the CGSM in the uncoupled approach

Regardless of the choice of optimization algorithm, the robustness of the design is evaluated in a post-design process. First, it is necessary to obtain different optimized designs by creating variations in the optimization algorithm. Figure 1 shows the CGSM flowchart, which is to be applied to each of the optimized designs. After two finite element analyses (FEA) with nominal parameters, the generalized forces are calculated, and the quantity of interest is obtained using the Castigliano theorem. As a result of the CGSM assumption, the internal strain energy of the truss can be calculated for any number of uncertain parameters using a Monte Carlo Simulation without further FEA. Finally, the mean value, standard deviation and c.o.v. of the quantity of interest are calculated. If the quantity of interest is the displacement, equations (4) and (5) are used.

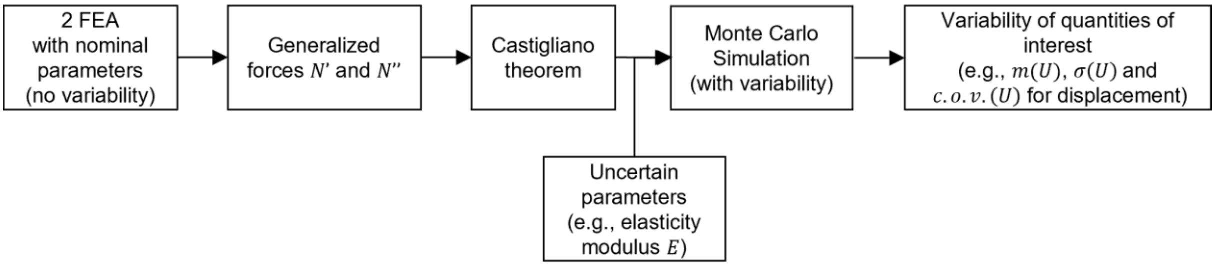

Figure 1 – Flowchart of the Certain Generalized Stresses Method (CGSM).

4. Example with the uncoupled approach

The example of the well-known MBB beam, shown in Figure 2, is studied using the uncoupled approach. The objective is to evaluate the influence of the uncertain elasticity modulus on the variability of the displacement at a point of interest P. Thus, the lower the observed variability of displacement, the more robust the structure. The uncoupled approach is a two-step process: first, various truss designs are obtained with an optimization algorithm, then second, the robustness of these structures is studied using the CGSM method.

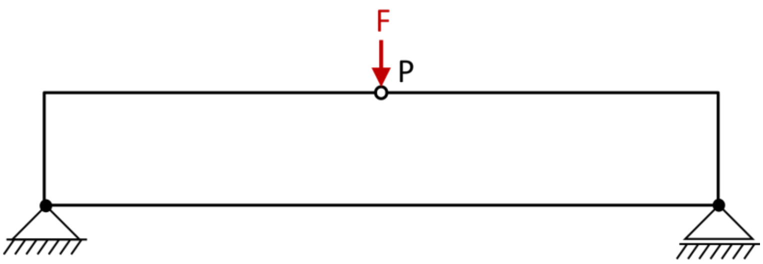

Figure 2 – Design domain load and boundary conditions of MBB beam.

For the optimization step, a ground structure-based layout optimization is used. The design domain is discretized with a regular grid of nodes. A fully connected ground structure is the configuration where each node is connected to all other nodes by discrete elements. These elements form the set of potential members, out of which the optimization algorithm identifies an optimal subset. The objective function minimizes the volume V of the structure:

$$\min V = \sum_{i=1}^{n} l_i A_i \tag{6}$$

where l is the length and A the cross-sectional area of the member i, and the cross-sectional areas are the optimization variables. Due to the discretization, the optimization result depends on the resolution of the grid of nodes.

Figure 3 shows six optimized truss structures for the minimum weight optimization of the MBB beam problem. The colors of the members correspond to the sign of axial force in the member. Members in red are under tension, while members in blue are under compression. All designs were obtained with the optimization tool LayOpt developed by Fairclough et al. [17]. Besides the primary ground structure-based layout optimization step, the tool allows subsequent geometry optimization and Heaviside simplification. For the optimization, a maximum compression to tension stress ratio of 1:2 was assumed, which roughly corresponds to the mechanical properties of a continuous carbon fiber composite. For a domain size of 24 x 4, designs with a volume between 43.41 and 44.44 were obtained. The difference between the lowest and highest volume is thus less than 2.4%. Simplifying, one can assume that all designs are "equally" optimum for the optimization objective defined in equation (6). However, it is noticeable that the number of members varies significantly between the designs and ranges from 23 to 249 members.

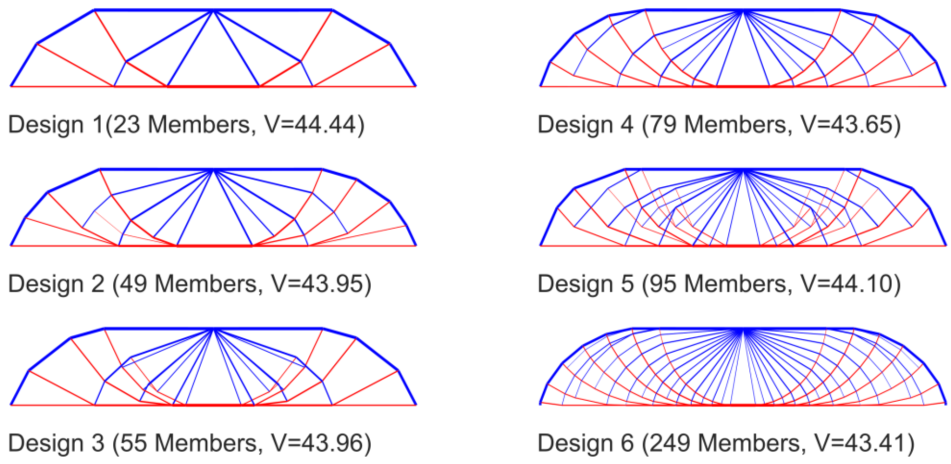

Figure 3 – "Equally" optimum truss designs of the MBB beam (the line thickness is proportional to the cross-sectional area).

The robustness evaluation step was conducted for each design using the CGSM. The displacement at the point of interest P was obtained following the described implementation (see section 3.3.). In the MBB beam example, the only external load is at the point of interest P in the direction of interest. Consequently, the force F in equations (2)-(4) is a physical force, but the axial force N' vanishes. Only one finite element analysis was needed to obtain the displacements for any configuration of uncertain parameters. This is an exceptional case as usually two finite element analyses are required for the CGSM.

The elasticity modulus of each member was defined as an uncertain parameter of the CGSM. Assuming a fully independent case of the uncertain parameters, the number of uncertain parameters was equal to the number of members in each design. The CGSM allows any statistical distribution of uncertain parameters. In the example, a truncated Gaussian distribution was used to define the statistical distribution of the elasticity modulus. The CGSM allows the calculation of the displacement variability for low to very high levels of variability of the elasticity modulus. In the MBB beam example, various levels of variability were considered with a c.o.v. from 5% to 30%. A c.o.v. of 30% represents an extremely high variability ranging from 10% to 190% of the nominal value. Standard software programs were used to perform the finite element analyses (Abaqus) and evaluate the displacement variability with the CGSM metamodel (Matlab). The quality of the CGSM was estimated using an error indicator (see section 3.1.). For ten trials and a c.o.v.(E) of 10%, errors of less than 0.3% were obtained for the mean value and standard deviation of the displacement. The low error indicator values show a high quality of the CGSM for the MBB beam example.

Almost identical mean values of displacement $m(U)$ at all variability levels were obtained for all six designs. However, the standard deviation differed clearly between the designs. Figure 4 shows the obtained variability of the displacement U for different levels of variability of the elasticity modulus E. A variability of displacement ranging from 1.3% to 2.8% was obtained for a c.o.v.(E) of 10%. For an extremely high level of variability or a c.o.v.(E) of 30%, the obtained variability of displacement ranged from 6.0% to 12.7%. The displacement variability thus differed by more than a factor of two between the “equally” optimum designs. Furthermore, a correlation between the number of members and the level of variability of the displacement was observed. A structure with more members is more robust to variability of the elasticity modulus than a structure with fewer members. This observation can be attributed to a compensation phenomenon that often occurs when the number of uncertain parameters increases. The design with the lowest displacement variability had 249 uncertain parameters, more than ten times than the design with the highest displacement variability.

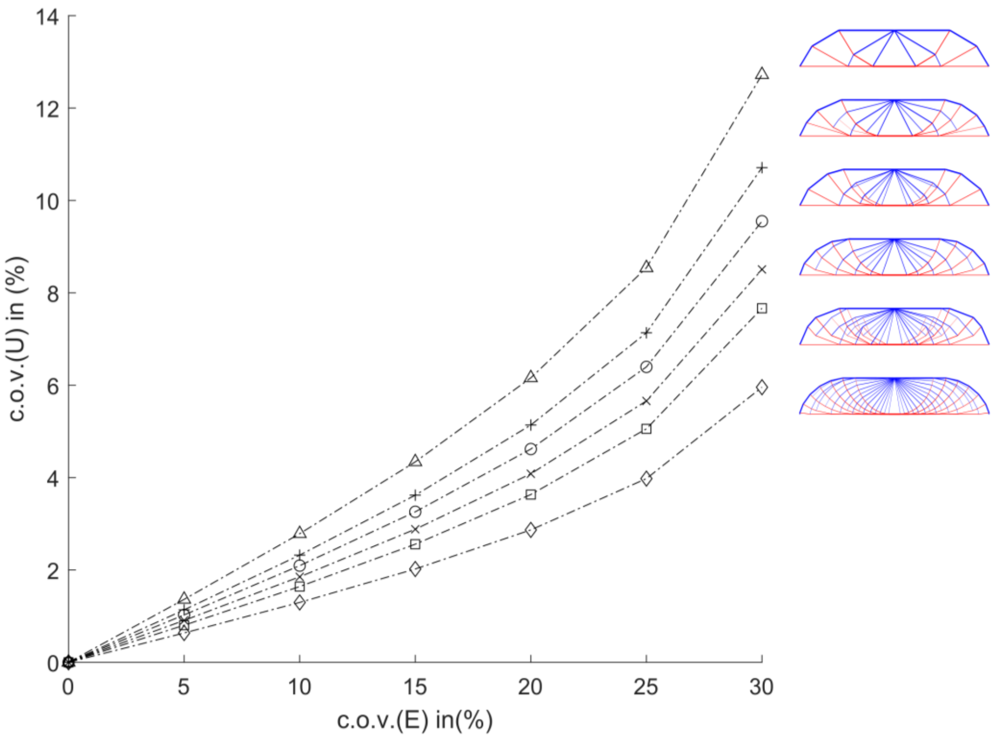

Figure 4 – Comparison of MBB beams: variability of maximum displacement U for several levels of variability of elasticity modulus E.

5. Conclusion and perspective

In this work, the evaluation of the robustness of optimized lightweight structures was discussed. Two different approaches were presented, namely, an uncoupled and a coupled approach. The uncoupled approach is a two-step method, where a robust design is selected from a set of optimized structures using stochastic modeling. This approach was developed independently from the optimization step. Using the well-known MBB beam optimization problem, it was demonstrated that lightweight optimization does not necessarily lead to robust structures. For manufacturing processes, such as Fused Filament Fabrication, where various sources for variability exist due to material and manufacturing, it is beneficial to consider robustness in the design process. The coupled approach may allow to obtain even more optimal robust designs than the uncoupled approach. In contrast to the two-step method, the design robustness is already considered during the design optimization step. However, the coupled approach leads to other challenges and requires the development of new and specific optimization techniques. Current research is addressing this coupled approach.

Références

[1] Eichenhofer, M.; Wong, J. C. H.; Ermanni, P. Continuous lattice fabrication of ultra-lightweight composite structures. Addit. Manuf., 18, 48–57, 2017.

[2] Feng, J.; Fu, J.; Lin, Z.; Shang, C.; Li, B. A review of the design methods of complex topology structures for 3D printing. Vis. Comput. Ind. Biomed. Art 1 (1), 5, 2018.

[3] Goh, G. D.; Yap, Y. L.; Agarwala, S.; Yeong, W. Y. Recent progress in additive manufacturing of fiber reinforced polymer composite. Adv. Mater. Technol., 4 (1), 1800271, 2019.

[4] Compton, B. G.; Lewis, J. A. 3D-printing of lightweight cellular composites. Adv. Mater., 26 (34), 5930–5935, 2014.

[5] Invernizzi, M.; Natale, G.; Levi, M.; Turri, S.; Griffini, G. UV-assisted 3d printing of glass and carbon fiber-reinforced dual-cure polymer composites. Materials, 9 (7), 2016.

[6] Brenken, B.; Barocio, E.; Favaloro, A.; Kunc, V.; Pipes, R. B. Fused filament fabrication of fiber-reinforced polymers: a review. Addit. Manuf., 21, 1–16, 2018.

[7] van de Werken, N.; Tekinalp, H.; Khanbolouki, P.; Ozcan, S.; Williams, A.; Tehrani, M. Additively manufactured carbon fiber-reinforced composites: state of the art and perspective. Addit. Manuf., 31, 100962, 2020.

[8] Kabir, S. M. F.; Mathur, K.; Seyam, A.-F. M. A critical review on 3d printed continuous fiber-reinforced composites: history, mechanism, materials and properties. Compos. Struct., 232, 111476, 2020.

[9] Li, S.; Xin, Y.; Yu, Y.; Wang, Y. Design for additive manufacturing from a force-flow perspective. Mater. Des., 204, 2021.

[10] Safonov, A. A. 3D topology optimization of continuous fiber-reinforced structures via natural evolution method. Compos. Struct., 215, 289–297, 2019.

[11] Hao, W.; Liu, Y.; Zhou, H.; Chen, H.; Fang, D. preparation and characterization of 3D printed continuous carbon fiber reinforced thermosetting composites. Polym. Test., 65, 29–34, 2018.

[12] Nath, P.; Olson, J. D.; Mahadevan, S.; Lee, Y.-T. T. Optimization of fused filament fabrication process parameters under uncertainty to maximize part geometry accuracy. Addit. Manuf., 35, 101331, 2020.

[13] Leary, M. Generative design. In design for additive manufacturing; Elsevier, 203–222, 2020.

[14] Knopf-Lenoir, C.; Lardeur, P.; Mandeng, R.; Wang, J. Minimisation de la variabilité du comportement statique des treillis. 10e colloque national en calcul des structures, Giens, France. 2011.

[15] Yin, Q.; Lardeur, P.; Druesne, F. Performances assessment of the modal stability procedure for the probabilistic free vibration analysis of laminated composite structures. Compos. Struct., 203, 474–485, 2018.

[16] Lardeur, P.; Arnoult, É.; Martini, L.; Knopf-Lenoir, C. The certain generalized stresses method for the static finite element analysis of bar and beam trusses with variability. Finite Elem. Anal. Des., 50, 231–242, 2012.

[17] Fairclough, H. E.; He, L.; Pritchard, T. J.; Gilbert, M. LayOpt: An educational web-app for truss layout optimization. Struct. Multidisc. Optim., 64 (4), 2805–2823, 2021.