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Expansion in structural dynamics : a perspective gained from success and errors in test/FEM twin building

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Résumé — Since tests only provide measurements at sensors, it is interesting to use models to estimate the response at all degree of freedom, correct measurement errors and possibly allow updating of model parameters. The paper gives an integrated perspective on methods developed by the control and structural dynamics communities and in particular methods seeking a trade-off between test and model error. The case of a measured brake squeal limit cycle is used to illustrate implementation details found to be important.
Mots clés — Shape expansion, model based estimation, model reduction, parameter updating

1 Introduction

Experiments provide an accurate representation of reality but are always spatially limited in the number of sensors. Tens of accelerometers, hundreds of scanning vibrometer points, thousands of camera pixels never give a full continuous field estimation : interior volume not measured, unreachable ares, hidden parts... This is referred to as the fact that measurements are spatially incomplete : Figure 1 left shows the gap between each measured points (with 3D-Scanning Laser Doppler Vibrometer) and the closest model surface ; all individual sensors are represented as red arrows on the middle and right figures.

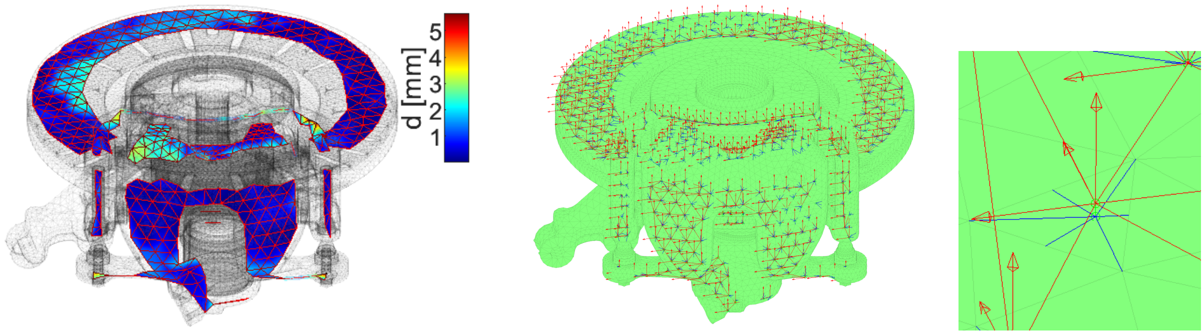

FIGURE 1 – Sample application of squeal limit cycle measurement on a brake : distance map between measured points and closest model surface (left), sensors as red arrows (middle) and zoom (right)

Models in general, and more specifically Finite Element Models for application in vibration of continuous structures, give a good understanding of many properties in very fine geometric detail but generally with biased estimates (frequencies and shapes are slightly off) and approximations (damping, poor representation of non-linearity, ...). Sensor measurements are related to model unknowns, called states or DOFs, defined in section 2.1, through observation equations detailed in section 2.2.

In the control community, *state estimation* methods, with Kalman filtering [1] being a common choice, clearly introduce the idea that model unknowns (called states or DOFs) can be estimated combining a somewhat wrong model and measured data. Provided that a good understanding of measurement errors exists, this estimate is actually more accurate than the measurement itself [2].

Expansion is the name given to state estimation by a number of authors of the structural dynamics community. Initial work quickly reminded in section 3.1 used the model to build a subspace : modal/SEREP [3, 4], static (based on Guyan reduction [5]), dynamic [6] and hybrid [7, 8].

Formulating a multi-objective problem combining measurement and model errors was rapidly identified as promising and led to expansion using the Error in Constitutive Relation [9, 10] and the Minimum Dynamic Residual Expansion [11, 12], that will be detailed in section 3.2.

The cost of solving this problem is however quite large for realistic FEM models and even recent attempts [13] find this to be a major problem without model reduction strategies discussed in section 4.1. The notion that model parameters can be adjusted as part of the estimation process is respectively referred to as Extended Kalman Filtering [1] or model updating. A minor distinction is that expansion is generally formulated in the frequency domain, when state estimation is generally expressed in time. But other than naming conventions, the differences really reside in the fact that for structural dynamics applications, model reduction is mandatory.

The test case used in section 4 to illustrate the expansion technique is a brake system for which squeal occurs during braking at about 4050Hz. A 3D Scaning Laser Doppler Vibrometer performs a sequential time measurement of all nodes in the test wireframe shown in figure 1 during the squeal and the procedure presented in [14] is used to extract the operating deflection shape (ODS) shown in Figure 2 left. The large associated model is used to show the efficiency of the reduction strategy, illustrate how expansion results allows a better analysis of the measured data and provides clues for further model updating and structural dynamics modifications.

2 States/DOF, observations/sensors, modes / shape functions

2.1 States/DOF, kinematic/model reduction

Properly defining states/DOF is the first need for estimation. In the general framework of continuous mechanics, one seeks to approximate the continuous solution using variable separation, in other words by assuming

$$u(x,t) = \sum_i T_i(x) q_i(t) = [T]\{q(t)\} \tag{1}$$

where DOF (Degrees Of Freedom) $q_i(t)$ are the amplitudes/coordinates associated with shape functions/basis vectors $T_i(x)$.

Since from (1) the solution lies within the subspace generated by shape functions, the same solution can be written using another basis of the same subspace. This amounts to changing DOF for the same response. Thus *DOF are arbitrary* and can only be properly defined in relation to a engineering choice of shape functions and several choices make sense.

In FEM models, *physical* DOF are typically obtained using polynomial shape functions that have unit amplitudes at a node with zero amplitude at all other nodes of an element.

In classical Component Mode Synthesis [5] (CMS), Guyan/static reduction achieves the same objective by selecting a subset of FEM DOF q_I to be *physical* and resolving the static enforced displacement problem leading to shape functions given by

$$\begin{Bmatrix} q_I(t) \\ q_C(t) \end{Bmatrix} = \begin{bmatrix} I \\ -K_C^{-1} K_{CI} \end{bmatrix} \{q_I(t)\} = \begin{bmatrix} T \end{bmatrix} \{q_I(t)\} \tag{2}$$

Modal DOF are defined by considering amplitudes associated with the mass normalized modeshapes of a reference elastic model.

$$\{q(t)\} = \left[\phi_{1\ldots NM}(x)\right] \{q_R(t)\} \tag{3}$$

In a more general setting, *kinematic reduction* is a generic process where one seeks solution within a restricted subspace built to satisfy model objectives. The choice of the subspace induces model reduction and should be chosen based on objectives. In FEM models, one chooses piecewise polynomials for their simplicity and with mesh refinement controlling the ability to have localized gradients. In CMS [5], one uses solutions to simple problems : eigenvalue computations, static responses leading to residual vectors, or even snapshots of some possibly non-linear solution [15]. Modes are used to control the model bandwidth and static shapes to control the ability to represent the effect of *representative* loads. In the choice of loads, external applied loads are obvious, but parametric loads have to be included for non-linearity or time varying models [16], and here sensor location loads will be introduced in section 4.1.

2.2 Sensors, observabillity / actuators, controllability

For structural dynamics application, one uses equations of motion of the form

$$[M(p)]\{\ddot{q}(t)\}+[K(p)]\{q(t)\}=[b]\{u(t)\}+\{F_{param}(p,q(t))\} \tag{4}$$

where $\{q(t)\}$ are *physical* DOF, p the model parameters whose nominal values define the nominal mass $[M(p_0)]$ and stiffness $[K(p_0)]$ matrices, $\{u(t)\}$ the inputs signals, $[b]$ the controllability matrix (spanning input signals to input forces at DOF) and $\{F_{param}\}$ represents modifications of the equations due to parameter changes, non-linearities, ... (these will be called parameteric forces later).

But this classical form lacks the idea of *sensors* which correspond to measurable quantities and in a model are related to DOF/states. In standard state space models, the system representation is decomposed in two parts

$$\begin{array}{ll}\{\dot{x}(t)\}=[A(p)]\{x(t)\}+[B(p)]\{u(t)\} & \text{Evolution equation}\\ \{y(t)\}=[C(p)]\{x(t)\}+[D(p)]\{u(t)\} & \text{Observation equation}\end{array} \tag{5}$$

which properly distinguishes physical measurements $\{y(t)\}$ which depend on how an experiment is built and states $\{x(t)\}$ which, as DOF, can be chosen arbitrarily.

To define sensors, one then builds the FEM observation equation of the form

$$\{y(t)\}=[c]\{q(t)\} \tag{6}$$

which, in the illustration of figure 1 right, accounts for the true position and orientation of vibrometer measurement. The linear observation formulation is adapted for all the typical modal analysis sensors : accelerometers, laser vibrometer, strain gauges and load cells, piezoelectric patches, ... The building of the observation matrix, requires placement of sensors in the FEM mesh which is called the topology correlation phase of experimental modal analysis [17].

The control matrix $[b]$ uses a similar process to represent spatial distribution of unit loads and reciprocity implies that b and c are transpose of each other for collocated inputs/outputs such that $\{u\}\{\dot{y}\}$ corresponds to power input.

3 Expansion or estimation of unknown states and possibly parameters

3.1 Subspace based method : interpolation, modal, static, dynamic

A class of expansion methods, called *subspace methods*, only use modeling information to select a subspace of possible displacements with dimensions inferior or equal to the number of sensors. Classical bases for subspace expansion are piecewise linear functions for wire-frame animation, modes [3, 4], unit responses at sensors computed either at 0 Hz (static expansion) or a target frequency (dynamic expansion [6]).

For $[T]_{N\times NR}$ a basis of this subspace, one assumes that the full displacement is of the form $\{q_{Exp}\}=[T]\{q_R\}$. The case where the subspace dimension is equal to the number of sensors

$$\{y_{Test}\}_{NS\times 1}=[c]_{NS\times NS}\{q_{Exp}\}_{NS\times 1} \tag{7}$$

can be used to illustrate the widespread confusion between *sensors* and *physical DOF*. If c is assumed non-singular, then $\{q_{Exp}\}=[c]^{-1}\{y_{Test}\}$ and one can use sensors as *physical DOF* by using $\{q\}\approx\left[[T][c]^{-1}\right]\{y_{Test}\}$ rather than the original $\{q\}\approx[T]\{q_{Exp}\}$. Another way to rephrase the same issue is to say that a model is defined by the kinematic subspace, while DOF are depend on the selection of a basis of this subspace.

This formulation of the problem illustrates the fact that enforcing motion at sensors can give differing results depending on the model reduction strategy : static shapes ignore inertia forces, which is wrong above the frequency of the the first fixed sensor mode ; dynamic responses at the measured frequency are optimal in the absence of noise, external or parametric forces ; modes may then lead to responses that are not relevant at that frequency [12].

In the presence of measurement noise, one may seek to smooth the response by considering a subspace of dimension smaller than the number of sensors and solving the least-squares problem

$$\{q_R\} = argmin \left\| \{y_{Test}\} - [c]\,[T]\,\{q_R\} \right\|_2^2 \tag{8}$$

The choice of the subspace is however then critical and the classic solution of using modes can give non-physical solutions.

3.2 Multi-objective problem combining test and FEM error

A more general class of methods, see among many [10, 11, 18], formulates expansion as a multi-objective minimization problem [19, 20] combining modeling and test errors and using the frequency information as in dynamic expansion. For a measured vector $\{y_{Test}\}$ and a model defined by parameters p, a state estimate/expanded vector $\{q_{Exp}\}$ is sought using the objective function

$$\begin{aligned} J(\{y_{Test}\},\{q_{Exp}\},p,\gamma) &= \|R_L(\{q_{Exp}\},p)\|_K^2 \quad + \gamma\ \|\{y_{Test}\} - [c]\,\{q_{Exp}\}\|_Q^2 \\ &= \varepsilon_{Mod}(\{q_{Exp}\},p) \quad + \gamma\ \varepsilon_{Test}(\{q_{Exp}\}) \end{aligned} \tag{9}$$

where

- $R_L(q_{Exp},p)$ is a *modeling error* residual, which depends on model parameters p and the expanded shape. Natural dynamic residual loads are $\{R_L\} = [Z(\omega,p)]\,\{q_{Exp}\} = \left[K(p) - \omega^2 M(p)\right]\{q_{Exp}\} =$ for modeshapes and $\{R_L\} = [Z(\omega,p)]\,\{q_{Exp}\} - [b]\,\{u(\omega)\}$ for frequency response to the harmonic input signal $\{u(\omega)\}$.
- $\|\|_K$ designates an energy norm. The motivation of this norm is explicit in the name Minimum Dynamic Residual Expansion [11] but is also motivated differently in the *Error in Constitutive Relation* work [10]. The classic norm first computes residual displacements induced by the residual loads $\{R_D\} = [K]^{-1}\{R_L\}$ and then evaluates the associated strain energy

$$\varepsilon_{Mod} = \frac{1}{2}\{R_D\}^H [K]\,\{R_D\} = \frac{1}{2}\{R_L\}^H [K]^{-1}\{R_L\} \tag{10}$$

- $\{y_{Test}\} - [c]\,\{q_{Exp}\}$ is the usual *test error* residual measuring difference between measurement and observation of the expanded shape
- $\|\|_Q$ designates a measurement error norm. Early work on the choice of Q assumed nothing and thus used an Euclidian norm : this is the commonly used norm by SDT

$$\varepsilon_{Test} = (\{y_{Test}\} - [c]\,\{q_{Exp}\})^H (\{y_{Test}\} - [c]\,\{q_{Exp}\}) \tag{11}$$

  Assuming gaussian measurement noise and thus relating Q to the noise variance is the usual approach in Kalman filtering. Bias is however often larger than variance so that other error characterizations may be used [18]. The use of *energy* metrics is mentioned in many papers, but one insists here on the fact that measurement errors are not related to a form of energy (in vibrometer noise they are related to optical signal losses).
- γ corresponds to the relative weight between the two objectives. The need to study this issue was identified very early [21] and has been the object of much attention since. The illustrations in section 4 will indicate that a log-scale search for the optimal value is needed from our experience.

The optimization process is practically performed in two stages :

- γ and p values are fixed so that $\{q_{Exp}\}$ and $\{R_D\}$ are solution of a linear problem of dimension $2\times N$. Model reduction needed to make this problem affordable is discussed in section 4.1.
- non-linear optimization is performed on γ and p :
  - γ is chosen to find a balance between test and FEM error as illustrated in section 4.2
  - when model updating is performed, optimal model parameters p minimize objective (9) as discussed in section 4.3

The outputs of the optimization are thus the expanded shape $\{q_{Exp}\}$, the modeling error residual $\{R_D\}$ which provides clues on location of modeling errors, the choice of the multi-objective weighting γ and possibly the set of updated model parameters p, that minimizes the objective.

4 A perspective gained from SDTools applications

To illustrate lessons learned from many applications, one uses a brake model and test from Hitachi-Astemo-France. The measured operational deflection shapes (ODS) characteristic of squeal is shown in Figure 2 left. The objectives of using MDRE in this study are :

- the estimation of a continuous field representing the ODS shape in the whole structure as shown on Figure 2 right
- the analysis of the test/FEM correlation by considering errors coming from both the test and the model side
- the prediction of system evolution for modifications of the geometry (local remeshing or thickness modifications)

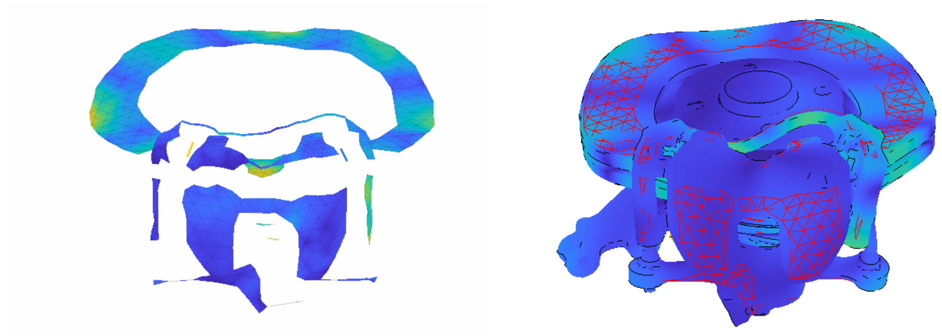

FIGURE 2 – Squeal ODS (left) and expansion result for $\gamma = 10^4$ (right)

4.1 Practical reduction strategies for expansion

The excessive numerical cost of solving (9) on a full model, see [13] for example, has strongly limited the spread of methods. Model reduction [12, 20] seems necessary for any practical application. While model reduction techniques are quite old in the Structural Dynamics community with modal techniques dating to Rayleigh, they are now also considered in many other fields and the presentation that seems to be becoming standard is to distinguish

- an offline, high cost, reduced order model building phase combining :
  - learning, where a data representative of the solution is obtained by a readily available algorithm
  - model building, where the data is reformated to obtain a reduced model that may be used at low cost
- an online usage phase where the low cost reduced order model is exploited, here to solve (9) or possibly dynamic expansion which is otherwise prohibitively expensive.

The learning phase, implemented in SDT [22], starts by combining some or all of the following vector sets

- modes of the nominal model $[\phi_{1:NM}(p_0)]$ truncated to the bandwidth of interest.
- parametric model enrichment, typically using the multi-model approach where learning modes is done at multiple design points [23]
- component modes can be used to access shapes at the component level which is very convenient when considering design changes or updating of components (see the CMT method in [24])
- residual vectors associated with unit load at sensor location [11]

$$[T_{Sens}] = [K]^{-1} [c]^T \tag{12}$$

which are computed using the classical approaches for residual vectors [5] which may contain mass shifting and prefiltering to avoid loads generating response on the low frequency modes. As the sensor density is large, it is possible to generate a basis with local support (values far away from the sensor are set to zero) thus generating a sparse reduction basis.

The second step of the learning phase is to build a basis from this collection of vectors using mass-orthonormalization and stiffness orthogonalization, using the nominal model $M, K(p_0)$ chosen in (4).

The brake system used as illustrations has 1.7 million DOF. The reduction basis contains the 100 first modes of the nominal model (up to 6000Hz as squeal occurs at 4050Hz) enriched with the static response to unit load through each of the 1293 sensors. A full reduced matrix would require 17 GB, but using the sparse sensor enrichment basis 1.2 GB is used. The learning phase (mode + sensor enrichment + orthonormalization) takes hour, while expansion, the online phase, for one γ using the reduced model then takes only a second. On top of model reduction, use of analytic gradient evaluations and reuse of factored matrices can vastly improve computational performance updating/optimization computations.

4.2 Choosing the γ weight and analyzing stress concentration at sensors

To analyze the influence of the objective weight on the expansion result, (9) must be computed for many γ values. Minimizing the objective function with a very low value of γ is equivalent to minimizing the modeling error residual alone which is easily found with $\{q_{Exp}\} = 0$ leading to $\varepsilon_{Mod} = 0$ and a maximum test error $\varepsilon_{Test} = \|y_{Test}\|_Q$. Increasing γ forces the expanded shape to more and more follow the test measurement up to maximum values of γ where $\varepsilon_{Test} = 0$ and the modeling error reaches a maximum. To display model and test error, whose evolution range with γ is huge, relative errors

$$\varepsilon^R_{Test} = \frac{\varepsilon_{Test}}{\|y_{Test}\|_Q} \; ; \; \varepsilon^R_{Mod} = \frac{\varepsilon_{Mod}}{\|q_{Exp}\|_K} \tag{13}$$

are shown in Figure 3 and relevant values are where the relative errors cross around 10%.

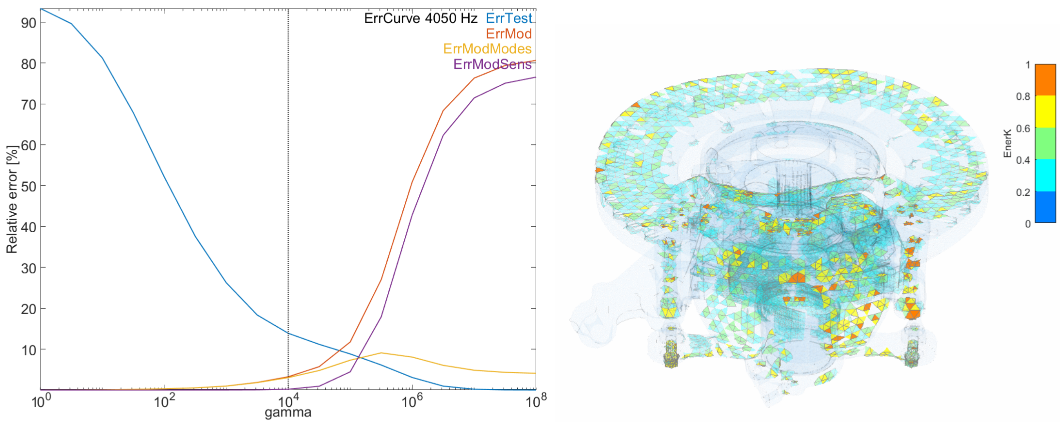

FIGURE 3 – Left : Evolution of relative errors with γ, right : expanded shape with stress concentration at sensor locations ($\gamma = 1e7$)

It is often useful to split the orthonormalized basis by block, distinguishing the part linked to global shapes $[\phi_M]$ and the remaining part linked to the sensor enrichment $\left[T^{\perp}_{Sens}\right]$

$$\{q\} = [T]\{q_R\} = \begin{bmatrix}\phi_M & T^{\perp}_{Sens}\end{bmatrix} \begin{Bmatrix} q_M \\ q_\perp \end{Bmatrix} \tag{14}$$

Using this decomposition and reminding that the associated basis is stiffness orthogonal, model error can be split into modal and sensor enrichment energies

$$\varepsilon_{Mod} = \{R_D\}|^T_M \begin{bmatrix} \ddots & & \\ & \omega^2_M & \\ & & \ddots \end{bmatrix} \{R_D\}|_M + \{R_D\}|^T_\perp \begin{bmatrix} \ddots & & \\ & \omega^2_\perp & \\ & & \ddots \end{bmatrix} \{R_D\}|_\perp \tag{15}$$

The modal energies, shown in Figure 3 left, become smaller than the sensor enrichment energies after $\gamma = 10^5$. This leads to an expanded shape with stress concentration close to the sensors illustrated in Figure 3 right, which is not representative of a physical stress field, and can thus not be used for model error localization or sensitivity studies.

4.3 Parameter changes : updating, structural dynamics modification

With an expanded modeshape, the next steps are to analyze the spatial distribution of test and modeling errors. Figure 4 left illustrates that the test error is almost only noise, with the remaining global motion being of the same order of magnitude as the noise level. This is typically seen as the indication of a good test and appropriate expansion. Figure 4 right, illustrates model error highlighting elements at bottom left and right of the structure close to the screws catching almost 50% of the error. This is used to orient model updating efforts towards changing the contact properties associated with screws and thus start the updating procedure. Note that in the plot, elements are sorted by energy density and grouped by blocks of 20%. In our experience, this specific choice notably helps interpretation for complex systems.

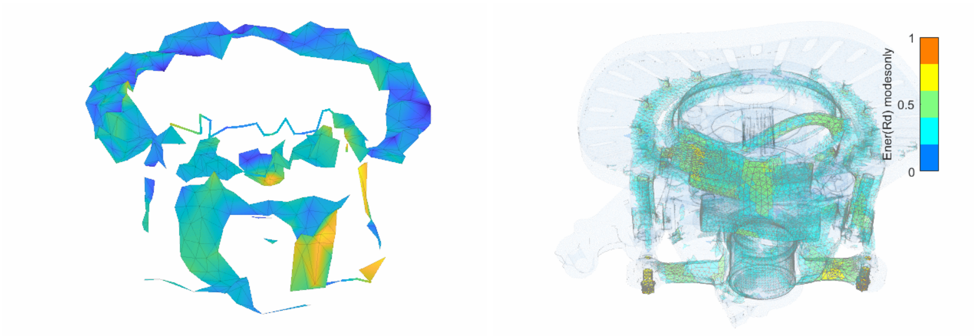

FIGURE 4 – Test error (left) and model error (right) repartition

5 Conclusion

The test case illustrated that the expansion is very practical for industrial models provided that model reduction is used to achieve the necessary performance and objective weighting is analyzed in detail to obtain a solution where stress distribution is properly reproduced. Introducing automated tools to analyze test errors and possibly changing the test error norm based on the results is a first perspective. Highlighting test errors to allow manual diagnostic of test is also important. Organizing the optimization process is a practical challenge for the industrial implementation of updating. For the case of squeal, the objective is often to modify the frequencies of modes involved in the limit cycle, and component mode tuning [24] can be very important.

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