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Size effect in phase-field fracture explained by the coupled criterion

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Abstract — We study the size effect introduced by the regularization length scale in phase-field fracture. To explain the observed mechanical phenomena, we compare the results to the coupled criterion. We show through simple examples that a correlation can be established between the regularization length, the stiffness, the toughness, and the material's strength. However, we emphasize that this correlation should be made based on a faulire surface rather than a single curve.
Mots clefs — Size effect, phase-field fracture, coupled criterion

1. Introduction

Fracture is one of the most feared failures in engineering. Therefore, design codes apply a significant safety factor to avoid it. Additionally, to the devastating consequences, the evolution of a crack is challenging to study in practice. Hence, predicting the initiation and propagation of a crack is of great importance for practicing engineers and scientists.

The first approach predicting brittle failure was proposed by Griffith [1]. He introduced a new measure to describe fracture toughness based on the energy release rate upon crack propagation. This model allowed us to describe well-known size effects in fracture. Later Bažant [2] realized that the description of Griffith is only valid if the crack is sufficiently large compared to the analyzed structure. Since then, many theories have been proposed to reproduce this elementary size effect in materials. Many theories assume that there is a transition zone where the stress singularity is somewhat regularized. Many of them assume that this is due to plastic activity.

In the early 2010s, Bourdin, Francfourt, and Marigo [3] proposed a variational approach, which diffuses the crack into the solid volume. This way, replacing the discrete crack representation with a continuous smeared one. This fundamental work gave rise to many papers where the originally proposed mathematical scheme was implemented and coupled with various physics. It was shown that this simple length scale parameter allowed the users to reproduce complex physical phenomena without adding other cumbersome criteria. Fig. 1 shows a few examples, what can be simulated using the phase-field method.

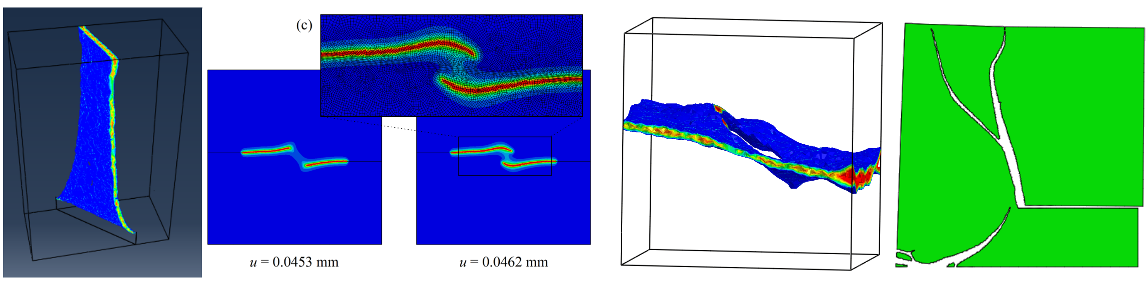

Figure 1 – From left to right: Mode I+III fracture; crack coalescence (reproduced from [4]); penny shaped crack; Kalthoff-Winkler test in PMMA

In this paper, we will discuss one particular aspect: the size effect introduced by cracks. We analyze different cases, focusing on critical loading, crack topology, and initiation dynamics.

2. Methods

The phase-field approximation replaces the discrete crack surface with a diffused one by introducing the crack density function. Thus the energy dissipated by the crack while propagating can be expressed as:

$$W(d) = g_c \Gamma \approx \int_\Omega g_c \gamma(d) d\Omega = \int_\Omega \frac{g_c}{2}\left(\frac{d^2}{l_c} + \frac{l_c}{2}|\nabla d|^2\right) d\Omega, \tag{1}$$

where Γ is the theoretical crack surface, γ is the crack surface density, d is the damage phase-field, g_c is the critical value of the energy release rate (a measure of toughness), and l_c is the regularization length scale. The damage variable d varies between 0 and 1. For undamaged solids its value is close to 0, while if a crack is fully formed $d = 1$.

By searching for the minimum of the following energy functional:

$$\mathcal{L} = \int_\Omega \psi^{el}(\mathbf{u}, d) d\Omega + W(d) + T(d) = \min, \tag{2}$$

the phase-field approach proposes a framework to solve fracture mechanics problems with variational approaches (*e.g.*, the finite element method). In eq. (2) $\psi^{el}(\mathbf{u}, d)$ is the degraded elastic strain energy and *T(d)* is a threshold function. More about the theory can be found in Ref. [4, 5].

3. Results

Progressively, through benchmark examples, the similarities, the differences, and the correlation between tensile strength and length scale are highlighted.

In the case of the simple extension (mode I) the energy release rate, the stress fields, and the crack path are all known analytically. Using the coupled criterion [6] the critical loading and the first unstable jump in the crack length can be determined. The critical loading as a function of the initial crack length is shown in Fig. 2. It can be seen that both phase-field and coupled criterion reproduces well the experimentally observed size-effect [2]. However, depending on the formulation the phase-field results converge to a different tensile strength when no crack is present.

The coupled criterion uses two parameters to describe the resistance of a structure: the critical energy release rate and the tensile strength.

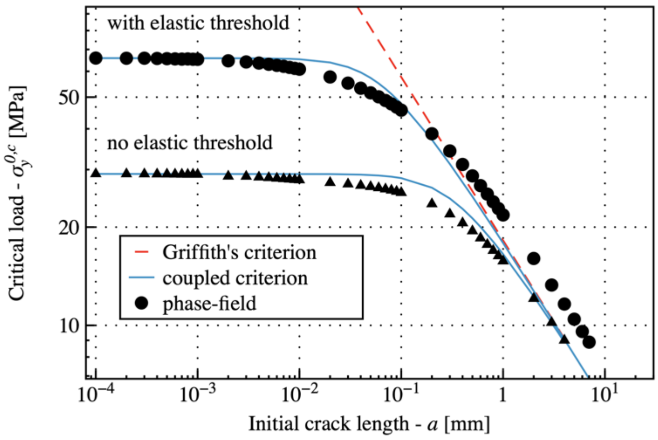

Figure 2 – Critical loading as a function of the initial crack length with different methods

These results allow us to establish the correlation between the length scale parameter used in phase-fields and the tensile strength in the coupled criterion.

Two additional test were carried out to investigate this correlation. We studied the initiation angle in simple shear and the unstable-stable propagation dynamics with a tapered double cantilever beam (TDCB) specimen. The detailed analytic and numeric calculations can be found in Ref. [7]. All the correlations are summarized in Fig. 3.

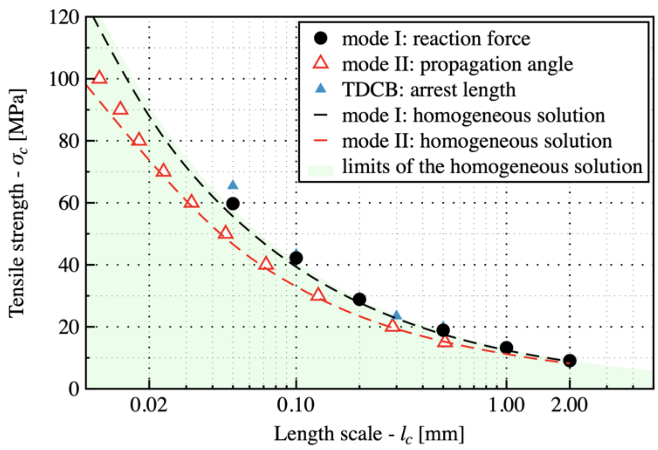

Figure 3 – Tensile strength as a function of length scale (l_c) for different cases without the threshold energy function (reproduced from Ref. [7])

It can be clearly seen that the correlation between length scale and tensile strength can be well defined, however this correlation is not unique. Rather than a single curve it can be described using a failure surface.

When the gradient term in eq. (1) is neglected, the differential equation is simplified and can be solved analytically. This is called the homogeneous solution of the phase-field fracture equation and can be expressed in the following for:

$$\sigma^c = \eta\left(\nu, \frac{\sigma_2}{\sigma_1}\right)\sqrt{\frac{Eg_c}{l_c}}\,, \tag{3}$$

where σ^c is the materials tensile strength, E is Young's modulus, ν is Poisson's ratio, and the function η takes into account the effect of the stress state.

This equation was plotted in Fig. 4.

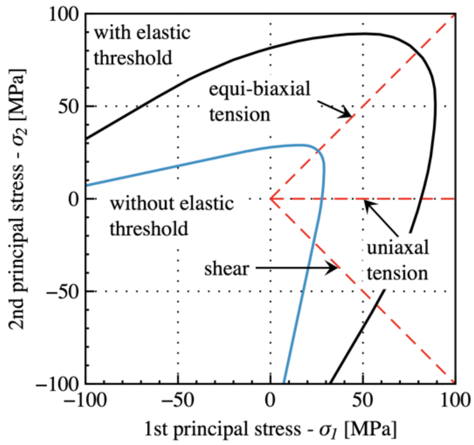

Figure 4 – Homogeneous phase-field solution for plane strain cases. Maximum tensile stress as a function of the principal stress state.

4. Conclusion

The paper presents a short overview of the correlation between the tensile strength and the regularization length scale used in phase-field calculations. We used the critical loading, crack topology, and the unstable initiation length to establish this correlation through a size effect present in most engineering materials. We found that the two quantities are related through a complex surface rather than a single curve. Now it is essential to develop an experimental way to quantify this regularization locally.

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